Printed Page:- 04 **Subject Code:- BMCA0105**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

MCA

SEM: I - THEORY EXAMINATION (2023-2024)

Subject: Discrete Mathematics

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION-A 20

1. Attempt all parts:-

1-a. What is the range of the function $f(x) = x^2 + 1$? (CO1) 1

(a) All real numbers

(b) All positive real numbers

(c) All non-negative real numbers

(d) All integers

1-b. If a function is onto, what can be said about its range? (CO1) 1

(a) The range is a subset of the domain

(b) The range is equal to the domain

(c) The range is a proper subset of the codomain

(d) The range is equal to the codomain

1-c. When there is no repetition of edges in the path, it is called a ____ path? (CO2) 1

(a) Simple

(b) Circuit

(c) Closed

(d) None of the above

1-d. In a _____ the degree of each and every vertex is equal.(CO2) 1

(a) regular graph

(b) point graph

(c) star graph

(d) euler graph

1-e. What is a homomorphism between two rings? (CO3) 1

(a) A function that preserves the ring operations
(b) A function that maps every element to the additive identity
(c) A function that maps every element to the multiplicative identity
(d) A function that maps the additive identity to itself

1-f. What is the quotient ring? (CO3) 1

(a) The set of all cosets of an ideal in a ring
(b) The set of all elements in the ring
(c) The set of all elements in the quotient field
(d) The set of all cosets of a subfield in a field

1-g. What is the implication of p implies q in propositional logic? (CO4) 1

(a) $p \rightarrow q$
(b) $p \Leftrightarrow q$
(c) $p \leftrightarrow q$
(d) p'

1-h. In propositional logic what is logical equivalence? (CO4) 1

(a) Two statements that have the same truth values for all possible truth assignments
(b) Two statements that are always true
(c) Two statements that are always false
(d) Two statements that contain only variables

1-i. In combinatorics , what is the binomial coefficient C(n, k) also known as? (CO5) 1

(a) Factorial coefficient
(b) Permutation coefficient
(c) Combination coefficient
(d) Multiplication coefficient

1-j. How many different ways are there to arrange the letters of the word "MISSISSIPPI"? (CO5) 1

(a) 34650
(b) 1260
(c) 1302
(d) 5040

2. Attempt all parts:-

2.a. If A = {1, 2, 3} and B = {3, 4, 5}, find $A \cap (A \cup B)$. (CO1) 2

2.b. Define a poset on the set K = {2, 4, 6} with the relation "divides." Check if it is a partial order and draw its Hasse diagram. (CO2) 2

2.c. Define a set K = {-1, 0, 1} and an operation * such that $a * b = a^2 + b^2$. Determine if (K, *) forms a group. (CO3) 2

2.d.	Show that statement $(x \Rightarrow y) \vee (y \Rightarrow x)$ is a tautology. (CO4)	2
2.e.	Given the recurrence relation: $a_n = 2a_{n-1} + 3$ with the initial condition $a_0=1$, find the value of a_3. (CO5)	2

SECTION-B 30

3. Answer any five of the following:-

3-a.	Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a + 1} is reflexive, symmetric or transitive.(CO1)	6
3-b.	Determine if the relation R = {(x, y) x + y = 10} is an equivalence relation. (CO1)	6
3-c.	Let Z = {m, n, o, p} with the relation R = {(m, m), (n, n), (o, o), (p, p), (m, n), (n, o), (o, p)}. Determine if R is a partial order and draw its Hasse diagram. (CO2)	6
3-d.	Consider the set I' = {1, 2, 4, 8, 16} with the relation "divides." Check if it is a partial order and draw its Hasse diagram.(CO2)	6
3.e.	Set G = { 1, ω, ω^2 } i.e., three roots of unity. Prove that G is an abelian group with operation multiplication. (CO3)	6
3.f.	Show that statements $((P \Rightarrow Q) \Rightarrow R)$ and $(P \Rightarrow (Q \Rightarrow R))$ are similar or not with the help of truth table. (CO4)	6
3.g.	A committee of 3 is to be chosen from a group of 7 people. In how many ways can this be done? (CO5)	6

SECTION-C 50

4. Answer any one of the following:-

4-a.	If f: R → R is defined by $f(x) = x^2 - 3x + 2$, find f(f(x)). (CO1)	10
4-b.	In a class of 60 boys, there are 45 boys who play cards and 30 boys who play carom. How many play: a) Both the games b) Cards only c) Carom only (CO1)	10

5. Answer any one of the following:-

5-a.	Determine the minimal and maximal elements in the poset $(P(\{a, b, c\}), \subseteq)$. (CO2)	10
5-b.	Define Kruskal's Algorithm with help of an example. (CO2)	10

6. Answer any one of the following:-

6-a.	Let H = Z/9Z with addition and multiplication defined modulo 9. Check if (H, +, *) is a ring. (CO3)	10
6-b.	Let F be a field. Consider the set F={0,1,2,3} with the operations Addition: Modulo 4 and Multiplication: Modulo 4. a) Verify whether F forms a field with the given operations. b) If F does not form a field, identify which field properties are not satisfied. (CO3)	10

7. Answer any one of the following:-

7-a.	There is a group of people, each with different hobbies. We have the following predicates: L(x): x likes playing sports.	10

M(x): x likes music.

Translate the following statements into predicate logic:

a) There is at least one person who likes both playing sports and music.

b) Everyone who likes playing sports also likes music.

c) There is someone who does not like either playing sports or music. (CO4)

7-b. Determine the truth value of $\forall x \exists y(P(x) \wedge Q(y))$ i) if $P(x)$ is true for all x and ii) $Q(y)$ is true for some y. (CO4) 10

8. Answer any <u>one</u> of the following:-

8-a. In a deck of cards, how many ways can you select 4 cards of the same color? (CO5) 10

8-b. A box contains 6 red balls, 4 blue balls, and 3 green balls. How many ways can you choose 2 red balls and 1 green ball? (CO5) 10